Int. J. Heat Mass Transfer. Vol. 22, pp. 1009–1020
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0017-9310/79/0701-1009 $02.00/0

FUEL SPRAY VAPORIZATION IN HUMID ENVIRONMENT

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(*Received* 10 *August* 1978 *and in revised form* 29 *October* 1978)

Abstract—The vaporization and transport of a monodisperse fuel spray in a cold, humid environment are analyzed, allowing for the possibility that the water vapor may condense, either heterogeneously at the droplet surface or homogeneously in the gas medium, as the spray interior is chilled through fuel vaporization. Results show that the associated condensation heat release is substantial and can significantly enhance the fuel vaporization rate, especially for the heterogeneous mode. It is further found that after initiation of vaporization the droplet temperature rapidly approaches a characteristic, constant value with heterogeneous condensation; this renders analytical solution possible. Potential complications caused by water condensation during spray experimentation are also discussed.

NOMENCLATURE

A_i, B_i, C_i,	Antoine vapor pressure constants;
A,	area;
$\hat{A}$,	A/A_0;
B,	transfer number, equation (7);
$\hat{C}$,	chamber function, equation (36);
C_p,	specific heat;
D,	mass diffusivity;
L_i,	specific latent heat of vaporization/condensation;
$\hat{L}_i$,	L_i/L_W;
L_H,	heat utilized for droplet heating per unit change in droplet mass;
$\hat{L}_H$,	L_H/L_W;
m_i,	mass flow rate;
m,	Σm_i;
$\hat{m}_i$,	$m_i/(4\pi\rho_g D_g r_s)$;
p,	pressure;
Q_c,	$1+\varepsilon_{Fc}(\hat{L}_F-1)$;
r,	radial distance;
$\hat{r}$,	r/r_s;
R,	r_s/r_{s0};
t,	time;
T,	temperature;
$\hat{T}$,	$C_p T/L_W$;
u,	velocity;
$\hat{u}$,	u/u_0;
W_i,	ratio of molecular weight of i to that of the noncondensable species;
x,	axial distance along chamber;
$\hat{x}$,	$x\rho_g D_g/(\rho_l u_0 r_{s0}^2)$;
X_i,	molar fraction;
Y_i,	mass fraction.

Greek symbols

ε_i,	m_i/m;
ρ,	density;
ω_F,	fractional amount of fuel vaporized, $(1-R^3)$;
ω_W,	fractional amount of water condensed.

Subscripts

c,	state corresponding to constant droplet temperature solution;
F,	fuel;
g,	gas phase;
i,	index for species;
l,	liquid phase;
s,	droplet surface;
W,	water;
0,	initial state.

Superscript

$*$,	fully vaporized or equilibrium state.

1. INTRODUCTION

THE RATE of combustion of a fuel spray is frequently controlled by the rate with which the individual droplets within the spray interior vaporize. In general the primary driving force to effect vaporization is the sensible heat of the hot environment within which the spray is introduced. Through conductive–convective heat transport the originally cold fuel droplets subsequently heat up and vaporize, producing fuel vapor which is mixed with the oxidizer gas to render combustion possible.

There is, however, another potentially important source of heat whose utilization can significantly enhance the spray vaporization rate. This arises from the possibility that during vaporization the spray interior may be chilled to such an extent that the water vapor present in the gas medium may condense and release the associated heat of condensation, which can then supplement the heat needed for fuel vaporization.

This condensation-enhanced vaporization is expected to be particularly beneficial for spray vaporization in a relatively cold, humid, environment, for example during carburetion of the automotive engine. For systems similar to this it has been found [1–3] that under most situations the spray interior rapidly becomes saturated with fuel vapor, leading to complete termination of vaporization. Since it is generally recognized that incomplete fuel vaporization during carburetion causes maldistribution and other deleterious heterogeneous combustion characteristics, the possible minimization of these heterogeneous effects through water vapor condensation is significant. Similar arguments can also be extended to the direct injection systems because the environment of the spray core is likely to be cold and fuel rich due to the slow rate of entrainment.

It is also reasonable to expect that the extent of augmentation in the fuel spray vaporization rate can indeed be substantial. This is because the air–fuel mass ratio for near-stoichiometric combustion of typical hydrocarbon fuels (e.g. gasoline) is usually a large number, around 15. Therefore for a sufficiently humid environment the moisture content is of the same order as the liquid fuel introduced. Furthermore the specific latent heat of vaporization of water is much larger than those of the hydrocarbon fuels. It is then obvious that the water vapor present in the environment does contain sufficient latent heat of condensation which when released can supply a significant portion of the energy needed for fuel vaporization. Finally, it is also of interest to note that since water is a major product of hydrocarbon combustion, the water vapor generated at the hot combustion zone may diffuse back to the cold vaporization region where it condenses.

In the following we shall substantiate and quantify the above concepts through a study of monodisperse spray vaporization in an environment *initially* containing some water vapor. It is important to stress that since energy is conserved and therefore not "free", the condensation heat we are exploiting has to be originally present in the system. Our anticipated potential benefits obviously would not exist for a system in which water has to be injected and vaporized in order to humidify the inlet system.

In the present study two models will be formulated to bracket the limiting behavior of spray vaporization in the presence of water condensation. In the fastest limit, which is studied in Section 2 and is termed the heterogeneous condensation mode, condensation of water vapor is assumed to occur at the surface of the fuel droplets which, being colder than the gas, are ideally suited as heterogeneous nucleation sites. Since in this limit the condensation heat release is directly supplied to the droplet, the droplet vaporization rate is expected to be maximally enhanced. In the slowest limit, which is studied in Section 3 and is termed the homogeneous condensation mode, condensation occurs within the gas medium at the saturation temperature. This implicitly assumes that there exist some heterogeneous nucleation sites (e.g. dust particles) in the gas other than the fuel droplets. In this limit the increase in the droplet vaporization rate is caused by an increase in the gas temperature relative to the case of no water condensation. Due to the large amount of air mass that needs to be heated, the extent of increase in the gas temperature, and hence the droplet vaporization rate, are not expected to be as significant as the heterogeneous mode. Finally, the transport of the spray in a quasi-one-dimensional chamber, with either mode of condensation, is formulated in Section 5.

2. HETEROGENEOUS CONDENSATION MODE

1. *General discussions*

In this limit condensation of water vapor is assumed to take place at the droplet surface. In order to ascertain the *maximum* effects of water condensation on fuel vaporization, it is also assumed that the droplet surface is not covered by the condensed water so that fuel can vaporize uninhibited. This is a reasonable assumption due to the fact that since water and oil generally do not mix, the condensed water will contract under surface tension and therefore offer minimum blockage of the droplet surface. Furthermore, since in realistic situations a relative velocity, and hence shear stress, exists between the droplet and the gas stream, it is reasonable to expect that these loosely attached water globules or caps tend to be stripped away from the fuel droplet if they are of any significant size.

It can be similarly argued that under the rare situation when the droplet temperature becomes less than the freezing point of water, 273 K, such that ice crusts are formed on the droplet surface, the system would still behave more like a vaporizing droplet rather than the physically-impossible case of a nonvaporizing, frost-encased sphere in the present environment which is warmer than 273 K.

An important quantity governing droplet vaporization is the temporal variation of the droplet temperature distribution, particularly its value at the surface, T_s, because it directly influences the amount of heat the droplet receives from the environment, the extent of droplet heating required, and the surface concentrations of the fuel and water vapor. An accurate determination [4, 5] of the droplet temperature distribution is difficult because it requires the knowledge of the internal circulatory motion responsible for convective transport. It has been found [6], however, that the bulk droplet vaporization characteristics are only minimally dependent on the detailed description of internal circulation. This is because as long as the massive surface layer is heated at approximately equal rates, the subsequent heating of the much lighter inner core constitutes only minor perturbations to the total heat budget at the droplet surface. Therefore for mathematical expediency we shall assume that the droplet temperature is spacially uniform but temporally varying, which is the simplest possible model allowing for droplet heating [7].

The problem of interest can then be stated as follows. At time $t = 0$ an ensemble of evenly-distributed pure fuel droplets of uniform radius r_{s0} and temperature T_{s0} start to vaporize within a stagnant gaseous environment characterized by its temperature T_{g0}, and fuel and water vapor mass fraction, Y_{Fg0} and Y_{Wg0}, respectively. The initial mass fraction of the liquid fuel is Y_{Fl0}. The subsequent behavior of this spray ensemble is determined by analyzing the vaporization of a single fuel droplet and the collective modification of the gaseous medium by all of them, as will be shown in the following.

2. *Fuel droplet vaporization with water condensation*

To serve as input to the spray analysis, and neglecting the small interference effects caused by the presence of neighboring droplets [8, 9], we are concerned herein with the spherically-symmetric vaporization of a single fuel droplet in an infinite expanse of gas which has temperature T_g and mass fractions of the fuel and water vapor Y_{Fg} and Y_{Wg} respectively. Since the states of the droplet and the environment change at rates much slower than the rate of gas-phase heat and mass transport in the vicinity of the droplet [5], the droplet vaporization process can be considered to be quasi-steady with slowly-varying boundary conditions. By further assuming that the specific heat C_p and the conductivity coefficients are constants, and that the gas-phase Lewis number is unity, the non-dimensional gas-phase diffusive–convective conservation equations for fuel vapor, water vapor, and energy can be respectively written as [10]:

$$\hat{m} Y_F - \hat{r}^2\, \mathrm{d}Y_F/\mathrm{d}\hat{r} = \hat{m}_F, \tag{1}$$

$$\hat{m} Y_W - \hat{r}^2\, \mathrm{d}Y_W/\mathrm{d}\hat{r} = \hat{m}_W, \tag{2}$$

$$\hat{m}(\hat{T} - \hat{T}_s) - \hat{r}^2\, \mathrm{d}\hat{T}/\mathrm{d}\hat{r} = -(\hat{m}_F \hat{L}_F + \hat{m}_W + \hat{m}\hat{L}_H), \tag{3}$$

where $\hat{m} = m/(4\pi\rho_g D_g r_s)$, $\hat{r} = r/r_s$, $\hat{T} = C_p T/L_W$, $\hat{L}_F = L_F/L_W$, $\hat{L}_H = L_H/L_W$, $m = m_F + m_W$, m_i is the mass vaporization rate of species i, L_i the latent heat of vaporization of i, L_H the amount of heat utilized for droplet heating per unit change in the droplet mass, r the radial distance, ρ the density, and D the mass diffusivity.

Integrating equations (1) to (3) from the droplet surface to the ambiance yields explicit expressions for the three parameters of interest:

$$\hat{m} = \ln(1+B) \tag{4}$$

$$\varepsilon_F = \frac{(Y_{Wg} - Y_{Ws})Y_{Fs} + (Y_{Fg} - Y_{Fs})(1 - Y_{Ws})}{(Y_{Fg} - Y_{Fs}) + (Y_{Wg} - Y_{Ws})}, \tag{5}$$

and

$$\hat{L}_H = \varepsilon_F(1 - \hat{L}_F) - \frac{(\hat{T}_g - \hat{T}_s)(\varepsilon_F - Y_{Fs})}{(Y_{Fg} - Y_{Fs})} - 1, \tag{6}$$

where

$$B = \frac{(\hat{T}_g - \hat{T}_s)}{\varepsilon_F \hat{L}_F + \varepsilon_W + \hat{L}_H}, \tag{7}$$

and $\varepsilon_i = m_i/m$ such that $\varepsilon_F + \varepsilon_W = 1$.

Further realizing that the phase change processes occur at rates much faster than the gas-phase transport rates such that the vapor at the droplet surface can be considered to be saturated, and that water and oil do not mix such that they vaporize independently of each other, the vapor concentrations of fuel and water at the surface can be related to the droplet surface temperature through, say, the Antoine vapor pressure relation [11],

$$X_{is} = \frac{1}{760p} \exp\left[A_i - \frac{B_i}{(T_s + C_i)}\right], \tag{8}$$

where p is the system pressure expressed in units of atmospheres, A_i, B_i and C_i are constants for a given substance [11], X_i is the molar fraction which, for the present system, is related to the mass fraction Y_i through

$$Y_i = \frac{X_i W_i}{1 + \sum\limits_i (W_i - 1)X_i}, \quad i = F, W, \tag{9}$$

and W_i is the ratio of the molecular weight of i to that of the non-condensable species (e.g. dry air).

It is worth emphasizing that because of the immiscibility of water and oil, and subsequently the independence of the vapor pressures on their relative liquid-phase concentrations, a small quantity of water present at the droplet surface is sufficient to saturate the surface layer with its vapor. Therefore the assumption that fuel vaporization is physically uninhibited by the small amount of water present is consistent with the use of the equilibrium vapor pressure relation equation (8).

Equations (4)–(9) show that $\hat{m}$, ε_F, and $\hat{L}_H$ are functions of the droplet temperature $\hat{T}_s$ and the states of the ambiance characterized by $\hat{T}_g$ and Y_{ig}. These will be subsequently determined through coupling with the liquid-phase heat balance and overall conservation relations for the spray.

Finally, it may be noted that since we expect the fuel to vaporize and water to condense, $\varepsilon_F \geqslant 1$ and $\varepsilon_W \leqslant 0$. Therefore in situations where water condensation has not yet taken place, then $\varepsilon_F = 1$, and equation (5) together with the water vapor pressure relation equation (8) are not needed.

3. *Droplet temperature transient*

By definition

$$L_H = \frac{(4\pi/3)r_s^3 \rho_l C_p \,\mathrm{d}T_s/\mathrm{d}t}{-\mathrm{d}[(4\pi/3)r_s^3 \rho_l]/\mathrm{d}t}, \tag{10}$$

which when expressed in non-dimensional form is

$$\hat{L}_H = -\mathrm{d}\hat{T}_s/\mathrm{d}(\ln R^3), \tag{11}$$

where $R = r_s/r_{s0}$. Therefore by equating equation (11) with the expression obtained through gas-phase energy conservation, equation (6), the rate of change of the droplet temperature as evaporation proceeds is given by

$$\frac{\mathrm{d}\hat{T}_s}{\mathrm{d}(\ln R^3)} = 1 - \varepsilon_F(1 - \hat{L}_F) + \frac{(\hat{T}_g - \hat{T}_s)(\varepsilon_F - Y_{Fs})}{(Y_{Fg} - Y_{Fs})}, \tag{12}$$

which is subject to the initial condition that $T_s = T_{s0}$ when $R = 1$.

4. *Spray conservation relations*

From energy conservation for the present adiabatic system, we have

$$\hat{T}_{g0} + Y_{Fl0}(\hat{T}_{s0} - \hat{L}_F) = (1 + \omega_F Y_{Fl0} - \omega_W Y_{Wg0})\hat{T}_g + (1 - \omega_F)\, Y_{Fl0}(\hat{T}_s - \hat{L}_F) + \omega_W Y_{Wg0}(\hat{T}_s - 1) \tag{13}$$

where $\omega_F = 1 - R^3$ is the fractional amount of liquid fuel that has vaporized, and ω_W is the fractional amount of water vapor that has condensed. By definition they are related to Y_{Fg} and Y_{Wg} through

$$Y_{Fg} = \frac{Y_{Fg0} + \omega_F Y_{Fl0}}{1 + \omega_F Y_{Fl0} - \omega_W Y_{Wg0}}, \tag{14}$$

$$Y_{Wg} = \frac{(1 - \omega_W) Y_{Wg0}}{1 + \omega_F Y_{Fl0} - \omega_W Y_{Wg0}}. \tag{15}$$

In writing equation (13) we have not accounted for the additional heat release as water freezes when the droplet temperature drops below 273 K. This amount is much smaller than the condensation heat release.

From conservation of water we can write

$$\frac{d(\omega_W Y_{Wg0})}{dt} = -\left[\frac{Y_{Fl0}}{(4\pi/3) r_{s0}^3 \rho_l}\right] m_W, \tag{16}$$

which states that the rate of depletion of the water vapor from the gas medium is equal to the rate of water condensation onto one droplet times the total number of droplets.

Similarly, conservation of fuel states that

$$\frac{d[(4\pi/3) r_{s0}^3 \rho_l \omega_F]}{dt} = m_F. \tag{17}$$

Dividing equation (16) by equation (17), we have

$$\frac{d\omega_W}{dR^3} = \left(\frac{Y_{Fl0}}{Y_{Wg0}}\right)\left(\frac{1}{\varepsilon_F} - 1\right), \tag{18}$$

which is to be solved subject to the initial condition $\omega_W = 0$ when $R = 1$. When $Y_{Wg0} \to 0$, $\varepsilon_F \to 1$; therefore the differential is finite.

This completes our formulation for the heterogeneous condensation mode. The final solution involves solving the coupled first order non-linear ordinary differential equations of equations (12) and (18), with the inhomogeneous terms being functions of ω_W, R, and $\hat{T}_s$. If water does not condense, then equation (18) is not needed.

5. *Criterion for complete spray vaporization*

Without solving equations (12) and (18), it is possible to derive an explicit criterion allowing *a priori* assessment of the ability of the spray to achieve total vaporization. Designating the fully vaporized state, $\omega_F = 1$, by the superscript "*", it is then required that $T_g^* > T_s^*$, $Y_{Wg}^* \geq Y_{Ws}^*$, and $Y_{Fs}^* > Y_{Fg}^*$. Therefore a sufficient, although not necessarily unique, criterion governing full vaporization can be obtained by equating any two of the above three relations such that the remaining inequality will serve as the criterion.

As an illustration we have adopted $T_g^* = T_s^*$, $Y_{Wg}^* = Y_{Ws}^*$, and $Y_{Fg}^* < Y_{Fs}^*$ as our criterion. Using equations (13)–(15), it can be shown that the single inequality to be satisfied for complete spray vaporization is

$$\frac{Y_{Fg0} + Y_{Fl0}}{1 + Y_{Fl0} - Y_{Wg0}} < \frac{Y_{Fs}^*}{1 - Y_{Ws}^*}, \tag{19}$$

where Y_{Fs}^* and Y_{Ws}^* are functions of $\hat{T}_g^*$ given by equations (8) and (9), and $\hat{T}_g^*$ is to be iteratively determined from the relation:

$$\hat{T}_g^*(1 + Y_{Fl0}) = \hat{T}_{g0} + Y_{Fl0}(\hat{T}_{s0} - \hat{L}_F) + [Y_{Wg0} - Y_{Ws}^*(1 + Y_{Fl0})]/(1 - Y_{Ws}^*). \tag{20}$$

6. *Results*

Equations (12) and (18) have been numerically integrated using the Runge–Kutta scheme. An *n*-heptane spray vaporizing in humid air, with $C_p = 0.35\,\mathrm{cal\,gm^{-1}\,K^{-1}}$, $L_F = 75.8\,\mathrm{cal\,gm^{-1}}$, $L_W = 590\,\mathrm{cal\,gm^{-1}}$, $W_F = 3.46$, $W_W = 0.622$, is adopted for study. The Antoine coefficients A_i, B_i and C_i, for heptane and water, are listed in [11].

To limit the extent of our parametric study, we shall also consider fixed values of $T_{s0} = 300\,\mathrm{K}$ and $Y_{Fg0} = 0$. The dependence of the system on T_{s0} and Y_{Fg0} is physically obvious and hence need not be specially considered. Results will be presented for an environment which initially is either fully humid, or half humid (on molar basis), or completely dry. Corresponding to $T_{g0} = 280\mathrm{K}$, 300K, and 320K, the saturation water contents are $Y_{Wg0} = 0.005979$, 0.02183, and 0.06722 for the fully-humid states, and 0.002984, 0.01084, and 0.03294 for the half-humid states.

Figures 1 and 2 show the variation of T_s with the fractional amount of liquid fuel vaporized, ω_F, which is an indication of the progress in the spray vaporization process. Since $T_{s0} = 300\mathrm{K}$ is higher than the wet bulb temperatures of heptane for the values of T_{g0} considered, it is seen that T_s generally decreases as vaporization is initiated; the only exception here is the fully-humid case of $T_{g0} = 320\mathrm{K}$, for which the extent of condensation heat release causes an increase in the droplet temperature. The importance of water condensation is further illustrated by the fact that for a given T_{g0}, curves representing different levels of initial humidity immediately branch off as condensation is initiated. For $T_{g0} = 280\mathrm{K}$, the half-humid curve coincides with the non-humid curve because condensation is not possible owing to the extremely low level of humidity in such a cold environment. For this case complete spray vaporization cannot be achieved; a state of equilibrium is reached with about 6% of the fuel still remaining in the liquid phase. However, for a fully-humid environment complete fuel vaporization can be attained through the additional condensation

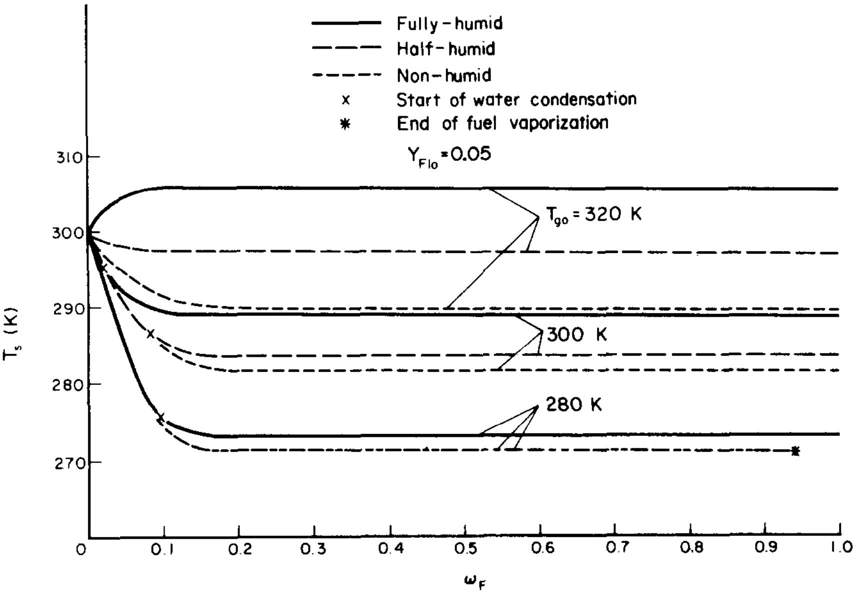

FIG. 1. Influence of initial gas temperature and humidity on droplet temperature variation for the heterogeneous condensation mode.

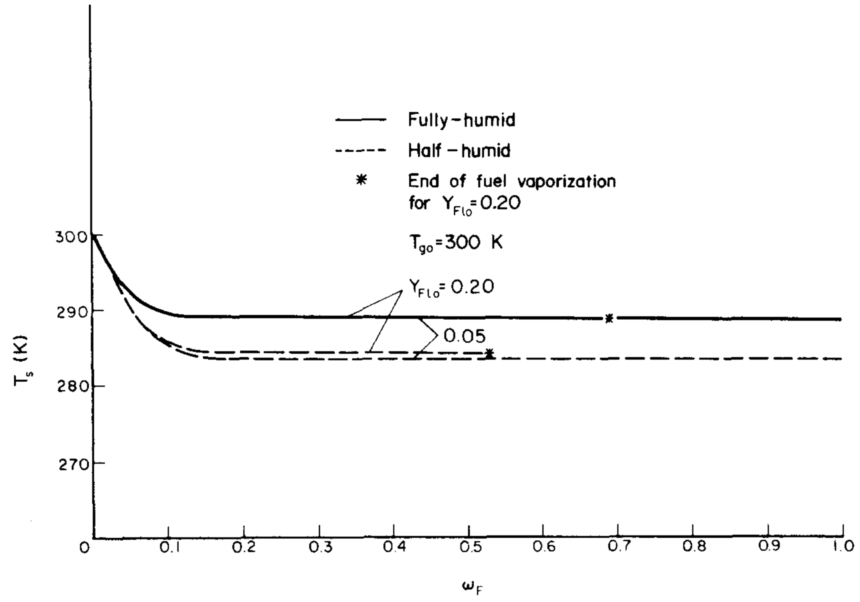

FIG. 2. Influence of initial liquid fuel concentration and humidity on droplet temperature variation for the heterogeneous condensation mode.

heat release. Therefore the potential of water condensation is particularly beneficial for cold mixtures which otherwise would not be able to achieve complete vaporization.

The present formulation assumes that the properties of the spray interior are uniform. In realistic situations, for example in the case of a spray jet, the cone region is much richer in fuel than the sheath region due to the finite rate of entrainment. To investigate effects due to this non-uniformity, Fig. 2 compares the variations of T_s for the near-stoichiometric and fuel-rich cases of $Y_{Fl0} = 0.05$ and 0.2 respectively. It is seen that whereas T_s depends only weakly on Y_{Fl0}, complete fuel vaporization for the rich case is very difficult to achieve. This is because there is now not only more fuel that needs to be vaporized, but there is also less air and hence correspondingly less sensible and condensation heat available for vaporization.

An interesting result shown in Figs 1 and 2 is that within 10–15% of the droplet lifetime the droplet temperature rapidly approaches a constant value. This behavior has also been observed in our previous studies [2, 3] on fuel spray vaporization in non-humid environments, and can be explained as follows. When the droplet ensemble vaporizes, the

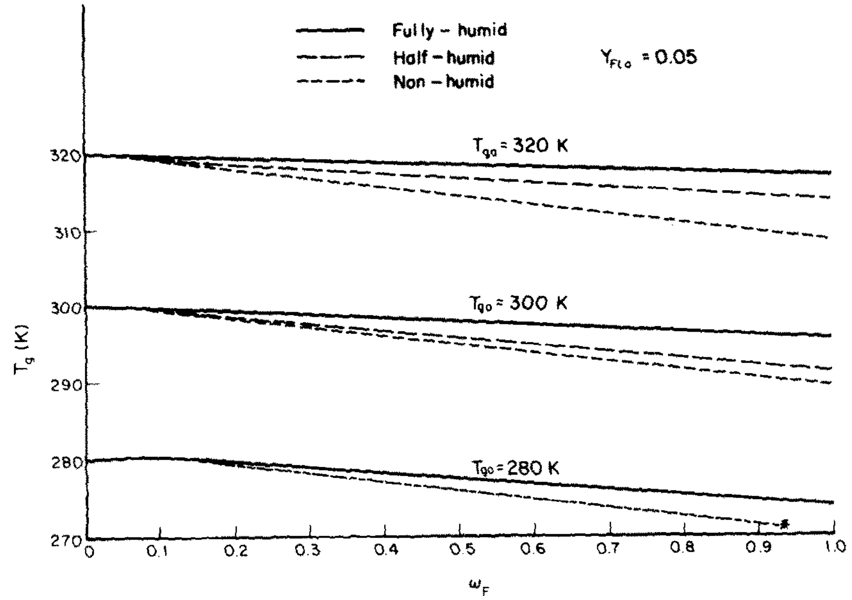

FIG. 3. Influence of initial gas temperature and humidity on subsequent gas temperature variations for the heterogeneous condensation mode.

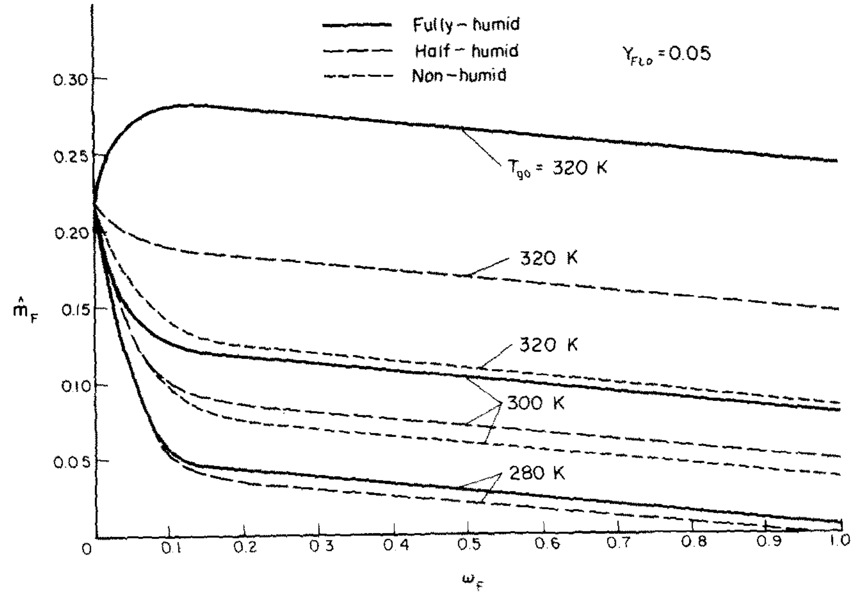

FIG. 4. Influence of initial gas temperature and humidity on fuel vaporization rate variation for the heterogeneous condensation mode.

gas medium is simultaneously chilled and enriched with fuel vapor. These two effects tend to suppress, and elevate, the droplet temperature in such proportions that the opposing trends cancel out. Therefore after the initial transient, the droplet attains a constant temperature. Similar argument can be extended to the present situation when there is the additional process of water condensation, which is just the reverse of vaporization.

It may also be emphasized that this constant droplet temperature is not the conventional wet-bulb temperature, which exists for droplet vaporization in a *constant* environment. In the present system the states of the environment, as well as the droplet vaporization rate, are continuously changing. The droplet temperature, however, is left unaffected. As will be subsequently shown, the establishment of this constant offers significant simplifications in the solution of the problem.

Figure 3 shows the variation of T_g as vaporization proceeds. During the initial 5–10% of the droplet lifetime T_g remains close to the initial values for those cases in which the droplet temperatures initially cool down. This is because even though their initial

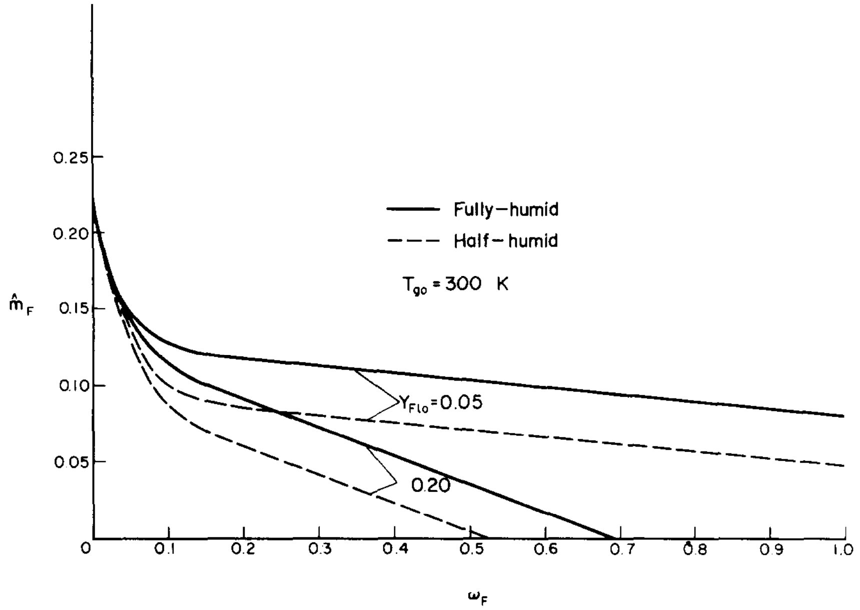

FIG. 5. Influence of initial liquid fuel content and humidity on fuel vaporization rate variation for the heterogeneous condensation mode.

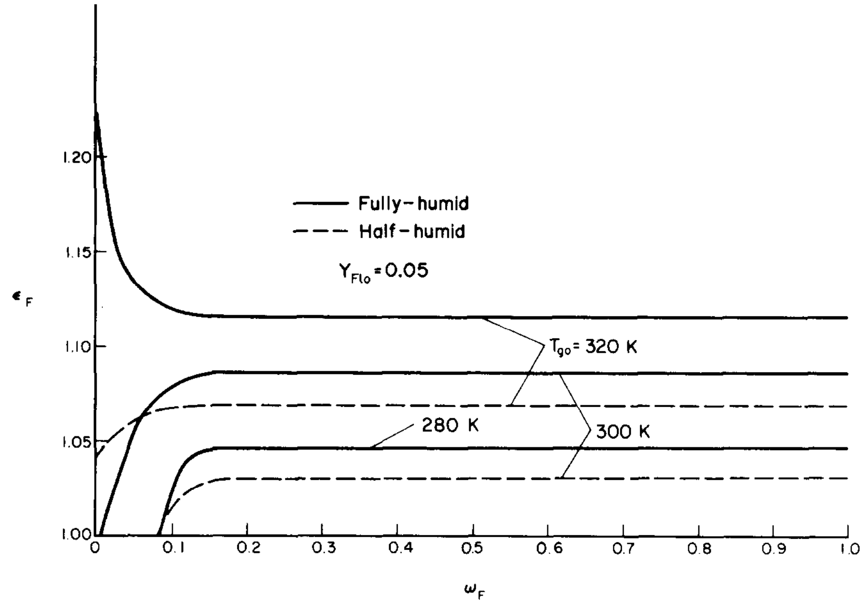

FIG. 6. Influence of initial gas temperature and humidity on the variation of fractional vaporization rate of fuel for the heterogeneous condensation mode.

vaporization rates are relatively fast, the latent heat of vaporization is supplied by the sensible heat of the "hot" droplet rather than the environment. In fact the excessive heat contained in the droplet may even produce an initial slight increase in T_g, as is found to be the case for $T_{g0} = 280$K. On the other hand if the droplet temperature initially increases, then the environment actively supplies heat to the droplet from the beginning and therefore T_g starts to decrease immediately. In all cases, however, it is interesting to note that after the initial transient T_g decreases almost linearly with ω_F.

Figures 4 and 5 show the fuel vaporization rate, $\hat{m}_F$, as functions of ω_F. The qualitative behavior is as expected. It is again of interest to note the almost linear dependence of $\hat{m}_F$ on ω_F after the initial transient. The most significant result shown in these figures, and indeed for the present study, is the extent of augmentation in the spray vaporization rate in the presence of water condensation. For example, Fig. 4 shows that for $Y_{Fl0} = 0.05$ at half-lifetime, a fully humid environment would enhance the fuel vaporization rate by about 30, 70, and 150% with $T_{g0} = 280$, 300, and 320K, respectively.

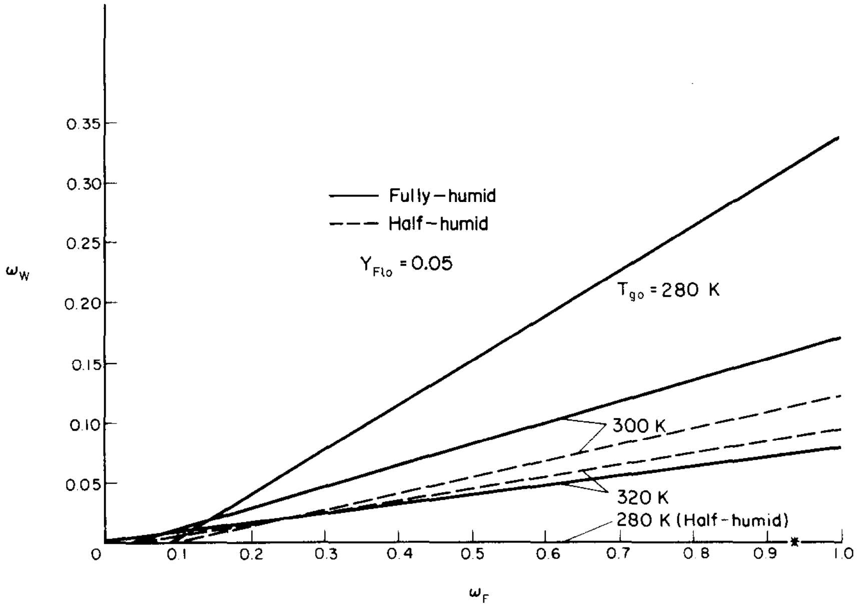

FIG. 7. Influence of initial gas temperature and humidity on the variation of the fractional amount of water vapor condensed for the heterogeneous condensation mode.

Figure 6 shows that for situations in which T_s, and hence $\dot{m}_F$, initially decrease, an initially increasing ε_F results. This is because as T_s decreases, the rate of water condensation increases. But since $\dot{m}_W$ is inwardly directed, the net mass flow rate, $\dot{m}$, is reduced. This causes an increase in $\varepsilon_F = \dot{m}_F/\dot{m}$. After the initial transient, ε_F assumes almost constant values for the rest of the vaporization process.

Figure 7 shows that after the initial transient the fractional amount of water vapor condensed varies almost linearly with the fractional amount of fuel vaporized, with slopes approximately being proportional to $(\varepsilon_F - 1)/Y_{Wg0}$ (equation 22). Therefore even though a relatively hot spray may have a higher rate of vaporization, and hence higher rate of condensation, it also contains more water vapor initially. Hence the net effect can be a smaller slope than the case of an initially cold spray.

7. *Constant droplet temperature solution*

The observation that the droplet temperature rapidly approaches a constant value after initiation of spray vaporization renders possible analytical solution by assuming the droplet is at this characteristic temperature, T_{sc}, throughout its lifetime. Hence by setting $d\hat{T}_s/dR^3 = 0$ in equation (12), $\hat{T}_{sc}$ can be iteratively determined from the implicit algebraic relation:

$$(Y_{Fg0} - Y_{Fsc})Q_c + (\varepsilon_{Fc} - Y_{Fsc}) \times [\hat{T}_{g0} + Y_{Fl0}\hat{T}_{s0} - (1 + Y_{Fl0})\hat{T}_{sc}] = 0, \quad (21)$$

where $Q_c = 1 + \varepsilon_{Fc}(\hat{L}_F - 1)$ and ε_{Fc} is determined from the following.

Assuming that ε_F is a constant, equation (18) can be readily integrated, yielding

$$\omega_W = \left(\frac{Y_{Fl0}}{Y_{Wg0}}\right)\left(1 - \frac{1}{\varepsilon_{Fc}}\right)\omega_F. \quad (22)$$

Substituting equation (22) into equation (5), and using equations (14) and (15), ε_{Fc} is given by

$$\varepsilon_{Fc} = \frac{Y_{Fg0}(1 - Y_{Wsc}) - Y_{Fsc}(1 - Y_{Wg0})}{(Y_{Fg0} + Y_{Wg0}) - (Y_{Fsc} + Y_{Wsc})}, \quad (23)$$

which is only a function of $\hat{T}_{sc}$, and hence shows that the assumption $\varepsilon_F = \varepsilon_{Fc}$ used in deriving equation (22) is consistent. Equation (22) also shows that ω_W varies linearly with ω_F, as is found in Fig. 7.

The transfer number B is now given by:

$$(1 + B) = \frac{(1 + B_c/Q_c)}{(1 + \omega_F Y_{Fl0}/\varepsilon_{Fc})}, \quad (24)$$

where

$$B_c = (\hat{T}_{g0} - \hat{T}_{sc}) + Y_{Fl0}(\hat{T}_{s0} - \hat{T}_{sc}). \quad (25)$$

For complete vaporization to occur, $B > 0$ at $\omega_F = 1$. From equation (24), the criterion for complete spray vaporization can now be simply stated as

$$B_c > Y_{Fl0}Q_c/\varepsilon_{Fc}. \quad (26)$$

If equation (26) is not satisfied, then vaporization terminates when the droplets reach the equilibrium size R, such that:

$$R = \{1 - (B_c\varepsilon_{Fc})/(Y_{Fl0}Q_c)\}^{1/3}. \quad (27)$$

Finally, using equations (4) and (24), it can be shown that for small values of Y_{Fl0}, the variation of the fuel vaporization rate as vaporization proceeds is

$$\frac{d\dot{m}_F}{d\omega_F} \simeq -Y_{Fl0}. \quad (28)$$

Equation (28) agrees with the numerical results of Figs 4 and 5, in which it is shown that after the initial transient, curves with different T_{g0} become parallel to each other, with a slope approximately equal to Y_{Fl0}.

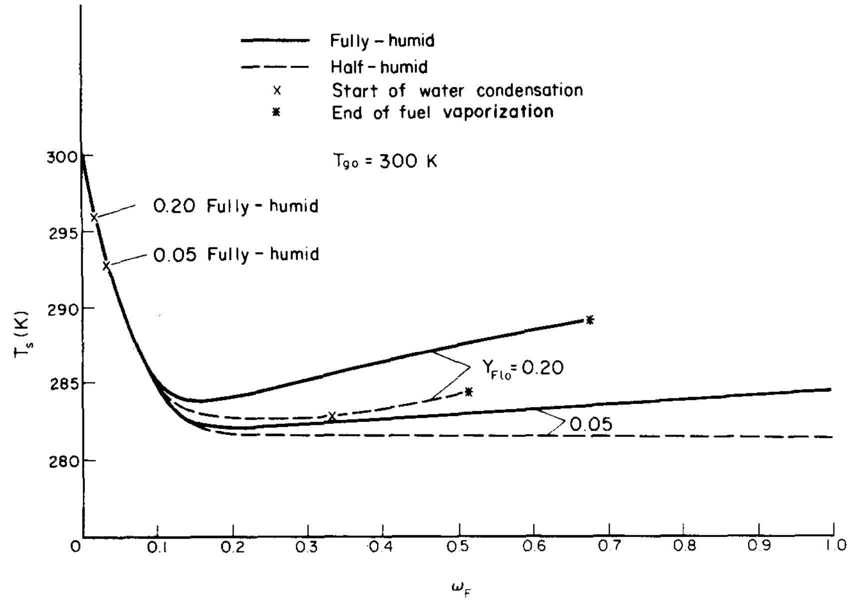

FIG. 8. Influence of initial liquid fuel concentration and humidity on droplet temperature variation for the homogeneous condensation mode.

The problem can therefore be considered to be solved completely analytically, with the only numerical work involved being the initial, iterative determination of T_{sc} from equation (21). Results obtained from the variable and constant droplet temperature solutions will be compared in Section 4.

3. HOMOGENEOUS CONDENSATION MODE

1. *Formulation*

In the slowest limit, condensation of water vapor occurs in the gas medium, either homogeneous or heterogeneous on some dust particles. Without being unduly concerned about the detailed nucleation and condensation processes, we shall simply assume that the water vapor is always in a saturated state in the gas medium, which implies that the excessive amounts will have been condensed, at the prevailing gas temperature, with the attendant condensation heat release.

Most of the formulation for the heterogeneous condensation mode remains valid here by setting $\hat{m}_W = 0$, and hence $\hat{m} = \hat{m}_F$, $\varepsilon_F = 1$, $\varepsilon_W = 0$, etc. The energy conservation relation, equation (13), is slightly modified to become

$$\hat{T}_{g0} + Y_{Fl0}(\hat{T}_{s0} - \hat{L}_F) = [1 + \omega_F Y_{Fl0} - \omega_W Y_{Wg0}]\hat{T}_g + (1-\omega_F) Y_{Fl0}(\hat{T}_s - \hat{L}_F) - \omega_W Y_{Wg0}. \quad (29)$$

The constraint on the saturation of the gas medium by water vapor is simply the Antoine relation evaluated at the gas temperature T_g, or

$$X_{Wg} = \frac{1}{760p}\exp\left[A_W - \frac{B_W}{(T_g + C_W)}\right]. \quad (30)$$

Furthermore, by definition

$$X_{Wg} = \frac{Y_{Wg}/W_W}{1-(1-1/W_W)Y_{Wg} - (1-1/W_F)Y_{Fg}}, \quad (31)$$

where Y_{Fg} and Y_{Wg} are given by equations (14) and (15), and ω_W is obtained from equation (29). Hence by equating equations (30) and (31), a transcendental equation

$$F(\hat{T}_g, \hat{T}_s, R^3) = 0, \quad (32)$$

results. The final solution then involves an iterative determination of $\hat{T}_g$ and $\hat{T}_s$ as functions of R^3 using equations (12) and (32). If the solution yields $\omega_W < 0$, then the environment has not saturated. Hence $\omega_W = 0$ and only equation (12) is needed.

A criterion for complete spray vaporization similar to that derived for the heterogeneous case can also be obtained for the present case.

2. *Results*

The system parameters adopted for illustration here are the same as those for the heterogeneous case. Figure 8 shows that for the fully-humid environment condensation is initiated rapidly after the excessive heat initially contained in the droplets has been dissipated through vaporization. However, for the half-humid environment initiation of condensation can be quite delayed. Indeed for $Y_{Fl0} = 0.05$ condensation fails to occur for the three initial gas temperatures investigated. The reason that condensation is difficult to start for the homogeneous mode is that an unsaturated gas medium has to be substantially chilled, through fuel vaporization, for water vapor to become saturated. This is particularly difficult to achieve when Y_{Fl0} is small. However, for the heterogeneous mode condensation is induced by the concentration gradient between the gas medium and the droplet surface. Since the droplet surface is usually at a lower temperature than the environment so that fuel vaporization is possible, the water vapor concentration at the surface is likely to be lower than

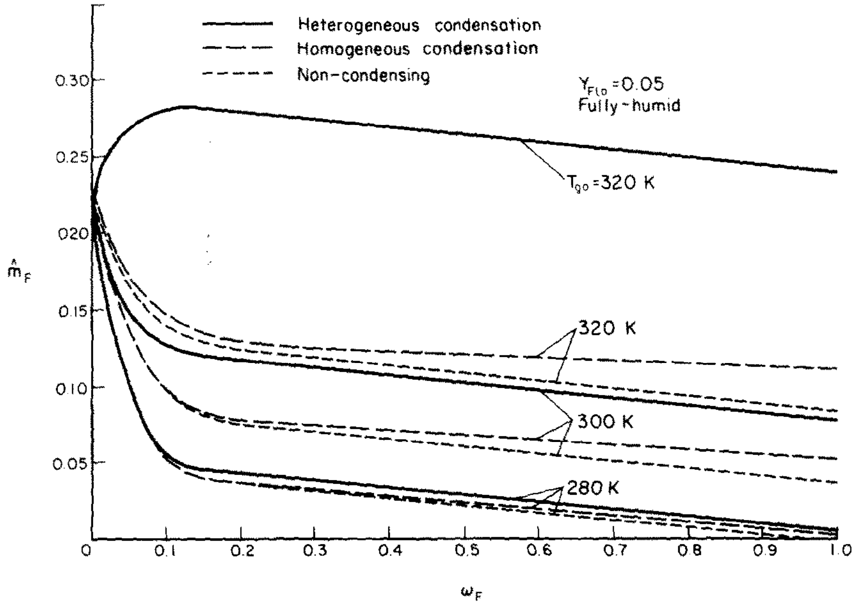

FIG. 9. Comparison of fuel vaporization rate variations for heterogeneous, homogeneous, and non-condensing modes for varying initial gas temperature.

that in the environment, rendering condensation possible.

Figure 8 also shows that in the presence of condensation, the droplet temperature increases almost linearly with ω_F after the initial transient. The behavior of other system parameters are qualitatively similar with those of the heterogeneous case and hence will not be elaborated. Finally, Fig. 9 compares the fuel vaporization rate, $\hat{m}_F$, in a fully-humid environment for the heterogeneous, homogeneous, and non-condensing modes. In realistic situations the vaporization rate will be bounded by the homogeneous and heterogeneous curves.

4. QUASI-ONE-DIMENSIONAL SPRAY TRANSPORT

In the following we shall consider the transport of the spray ensemble in an adiabatic, quasi-one-dimensional chamber with cross-sectional area $A(x)$, where x is the axial distance. To focus our attention on effects due to heat and mass transfer, we shall also assume that there is no velocity lag between the gas and the droplets.

Using the definition of m_F and the identity d/dt = ud/dx, where u is the velocity, we have

$$R\hat{u}\,\mathrm{d}R/\mathrm{d}\hat{x} = -\varepsilon_F \ln(1+B), \tag{33}$$

where $\hat{u} = u/u_0$ and $\hat{x} = x\rho_g D_g/(\rho_l u_0 r_{s0}^2)$. Furthermore, continuity requires that

$$\hat{u}\hat{A} = 1+\omega_F Y_{Fl0} - \omega_W Y_{Wg0}, \tag{34}$$

where $\hat{A} = A/A_0$.

Substituting equation (34) into equation (33) and integrating, we have

$$C(\hat{x}) = \int_R^1 \frac{(1+\omega_F Y_{Fl0} - \omega_W Y_{Wg0})}{\varepsilon_F \ln(1+B)} R'\,\mathrm{d}R', \tag{35}$$

where $C(\hat{x})$ is the chamber function defined as

$$C(\hat{x}) = \int_0^{\hat{x}} \hat{A}(x')\,\mathrm{d}x', \tag{36}$$

which is a measure of the axial distance travelled.

Figure 10 shows $C(\hat{x}^*)$ as a function of the inlet gaseous temperature T_{g0} for the heterogeneous, homogeneous, and non-condensing modes, where $\hat{x}^*$ is the location of either complete or equilibrium fuel vaporization. It is obvious that the shortening of the chamber length required to achieve complete spray

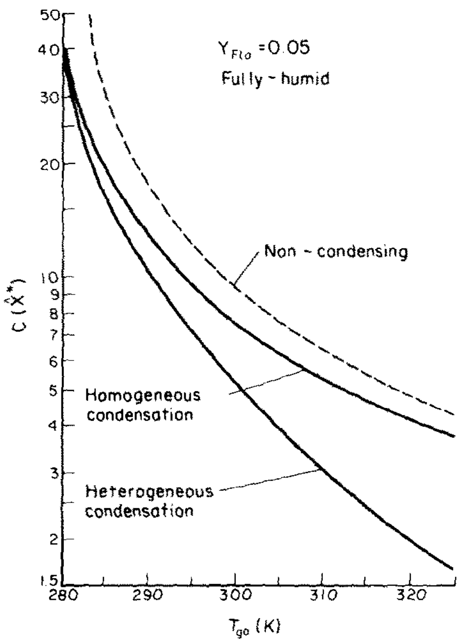

FIG. 10. Comparison of the chamber function for complete fuel vaporization for the heterogeneous condensation, homogeneous condensation, and non-condensing modes.

vaporization as a result of condensation is significant, particularly for the heterogeneous mode and at higher inlet temperatures.

By further invoking the constant droplet temperature assumption for the heterogeneous condensation mode, equation (35) becomes

$$C(\hat{x}) = \int_R^1 \frac{(1+\omega_F Y_{Fl0}/\varepsilon_{Fc})}{\varepsilon_{Fc} \ln[(1+B_c/Q_c)/(1+\omega_F Y_{Fl0}/\varepsilon_{Fc})]} \times R'\,\mathrm{d}R'. \quad (37)$$

Equation (37) can be integrated by assuming that the spray is not rich ($Y_{Fl0} \ll 1$) and the evaporator is sufficiently efficient ($Y_{Fl0} \ll B_c/Q$), yielding

$$C(\hat{x}) = \hat{m}_{Fc}^{-1}[0.5(1+z)(1-R^2) - 0.2z(1-R^5)], \quad (38)$$

where

$$\hat{m}_{Fc} = \varepsilon_{Fc} \ln(1+B_c/Q_c) \quad (39)$$

and

$$z = (1/\varepsilon_{Fc} + 1/\hat{m}_{Fc}) Y_{Fl0}. \quad (40)$$

Therefore in order to achieve complete spray vaporization, the chamber length needs to have a minimum length:

$$C(\hat{x}^*) = \hat{m}_{Fc}^{-1}(0.5+0.3z). \quad (41)$$

Figure 11 compares $C(\hat{x}^*)$ given by the exact expression equation (35), the constant temperature expression equation (37), and the efficient evaporation limit of equation (41). It is found that results from the constant droplet temperature integral agree with those from the exact integral to within 5% and hence can be considered to be accurate. Results from the efficient evaporation limit agree well with the exact results at higher inlet temperatures, although the agreement deteriorates as T_{g0} decreases and completely breaks down near the fuel saturation limit, as should be.

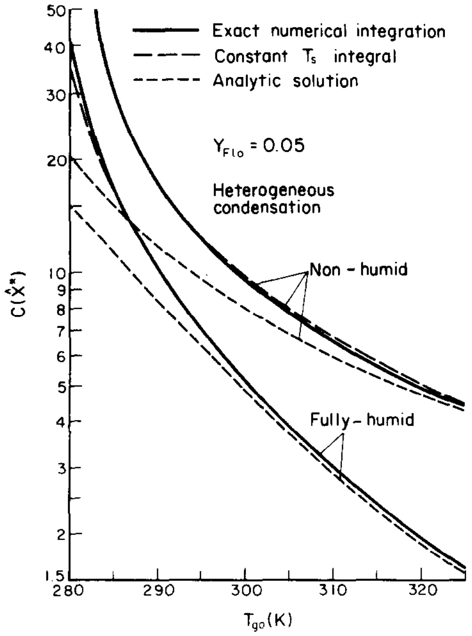

FIG. 11. Comparison of the accuracy of different solutions in predicting the chamber function for complete fuel vaporization for the heterogeneous condensation mode.

5. CONCLUDING REMARKS

In this paper we have investigated, and bracketed, the effects of water vapor condensation during the vaporization of a fuel spray in a humid environment. It is demonstrated that for the heterogeneous mode the effect of condensation is indeed significant and can substantially hasten the fuel vaporization process. For the homogeneous case the effect is relatively smaller because of the large amount of air mass that absorbs the condensation heat release. For both modes the potential of water condensation in enabling an initially cold spray to achieve complete vaporization, and therefore minimizing the deleterious heterogeneous combustion effects, are particularly significant.

The present results also indicate that caution should be exercised when experimentally measuring spray properties. The condensation process not only introduces an additional heat source, but can also give a larger value of the fuel droplet size in the presence of heterogeneous nucleation, and a larger value of the droplet number density in the presence of homogeneous nucleation. In both cases inaccurate data will be taken on the spray vaporization rate, the droplet distribution function, the total amount of fuel available for combustion, and its relative amounts in the gas and liquid phases. The difficulty in avoiding this masquerading effect is further compounded by realizing that even if the gas mixture is dried before using, water vapor can still be generated in the combustion region and possibly diffuse back to the vaporization region. It seems that a necessary precaution to take in ascertaining the accuracy of the data, is to ensure that the data do not violate the requirement of atom or species conservation.

Finally, considerations should also be given to gaseous experiments in which seedings are introduced for optical measurement purposes. These seedings may serve as heterogeneous nucleation sites and hence may become larger than assumed. Subsequently their velocity may not always be in phase with that of the gas, and therefore yielding inaccurate data on the gas velocity.

Acknowledgement—This work was sponsored by the Office of Basic Energy Sciences, Department of Energy, under Contract No. EG-77-S-02-4433.

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VAPORISATION D'UN BROUILLARD DE COMBUSTIBLE DANS UN ENVIRONNEMENT HUMIDE

Résumé—On analyse la vaporisation et le transport d'un brouillard monodispersé de combustible dans un environnement froid et humide, en admettant que la vapeur d'eau puisse se condenser soit de façon hétérogène à la surface des gouttes, soit de façon homogène dans le milieu gazeux, lorsque le coeur du brouillard est refroidi par la vaporisation du combustible. Des résultats montrent que la condensation associée peut sensiblement accroitre le taux de vaporisation du combustible, spécialement par le mode hétérogène. On trouve qu'après l'initiation de la vaporisation la température des gouttes approche rapidement une valeur constante caractéristique de la condensation hétérogène; ceci rend possible une solution analytique. On discute aussi des complications potentielles causées par la condensation de l'eau pendant l'expérimentation.

DIE VERDAMPFUNG VON EINGESPRITZTEM BRENNSTOFF IN EINER FEUCHTEN UMGEBUNG

Zusammenfassung—Es werden die Verdampfung und die Bewegung eines mono-dispersen Brennstoffsprühnebels in einer kalten, feuchten Umgebung untersucht, wobei die Möglichkeit berücksichtigt wird, daß der Wasserdampf entweder heterogen an der Tröpfchenoberfläche oder homogen in Gasmedium kondensiert, in dem sich der Nebel durch die Brennstoffverdampfung abkühlt. Die Ergebnisse zeigen, daß die Wärmeabgabe durch Kondensation von großer Bedeutung ist und die Verdampfungsgeschwindigkeit des Brennstoffes wesentlich verbessern kann, besonders im heterogenen Fall. Es stellte sich ferner heraus, daß nach Einsetzen der Verdampfung die Temperatur der Tröpfchen bei heterogener Kondensation sehr schnell einen charakteristischen konstanten Wert annimmt; dies läßt eine analytische Lösung möglich erscheinen. Mögliche Schwierigkeiten durch Wasserkondensation bei den Einspritzversuchen werden ebenfalls diskutiert.

ИСПАРЕНИЕ РАСПЫЛЕННОГО ТОПЛИВА ВО ВЛАЖНОЙ ОКРУЖАЮЩЕЙ СРЕДЕ

Аннотация — Анализируются процессы испарения и переноса монодисперсного распыла топлива в холодной влажной окружающей среде с учётом возможной конденсации водяного пара или гетерогенно на поверхности капли или гомогенно за счёт испарения топлива. Результаты показывают, что величина выделяющейся теплоты конденсации является значительной и может существенно усилить интенсивность испарения топлива, в особенности в режиме гетерогенной конденсации. Найдено, что начиная с момента возникновения испарения, температура капли быстро приближается к постоянному значению, характерному для процесса гетерогенной конденсации, что позволяет получить аналитическое решение. Также проведено обсуждение возможного усложнения задачи за счёт конденсации водяного пара.